

Bosonization of Massless Kalb-Ramond Fields in the presence of Fermions: A Path-Integral study

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Abstract

We present an exactly soluble, four-dimensional path-integral Bosonization procedure for dynamical fermions interacting through a spin-orbit coupling with abelian Kalb-Ramond Gauge fields.

In the last years, Kalb-Ramond field theory has been widely studied as an alternative dynamical quantum field scheme to the Higgs mechanism [1], as well as in relation to the dynamics of strings in the problem of string representation for Q.C.D. at large number of colors [2]. The basic formalism used to analyze such Kalb-Ramond non-perturbative quantum dynamics has been the Path-integral formalism, which has shown itself to be a very powerful procedure to understand correctly the different phases of the associated Kalb-Ramond Quantum Field Theory [3].

One important problem in those Path-integral studies, still missing in the literature, is that one related to the presence of interacting dynamical fermions in the Kalb-Ramond Gauge theory. In this letter, we shall describe the extension of previous path-integral dualization-bosonization studies [3] to the case of Fermionic matter coupling through a spin-orbit field quantum interaction.

Let us start by considering the Abelian Kalb-Ramond first order action but now in the

presence of dynamical fermions in the four-dimensional Euclidean world.

$$S[H, B, \psi, \bar{\psi}] = \int_{R^4} d^4x \left\{ \frac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - \frac{1}{6} H^{\lambda\mu\nu} \partial_{[\lambda} B_{\mu\nu]} + \bar{\psi} (i \not{\partial} + ie\gamma^\alpha \gamma^\beta \gamma^\mu H_{\alpha\beta\mu}) \psi \right\}. \quad (1)$$

Here the dynamical fields are the independent three-form H , the KR gauge field B and the Dirac fermion fields $(\psi, \bar{\psi})$.

We shall apply the bosonization procedure in the path-integral framework through the following theory's generating functional (normalized to unity) ([4]).

$$\begin{aligned} Z[J, \eta, \bar{\eta}] &= \int D^F[H] D^F[B] D^F[\psi] D^F[\bar{\psi}] \\ &\times \exp\{-S[H, B, \psi, \bar{\psi}]\} \\ &\times \exp\left\{-\frac{i}{2} \int_{R^4} d^4x (\bar{\eta}\psi + \bar{\psi}\eta + JB)(x)\right\}. \end{aligned} \quad (2)$$

It is worth call the reader attention that the Path-integral eq.(2) is invariant under the KR gauge symmetry, provide the external source corrent $J_{\mu\nu}$ is chosen to be divergence free and due to our proposed action term related to the direct interaction of the quantum fermionic matter with the Kalb-Ramond gauge field through its strenght three-form H – the spin orbit fermion interaction. (see eq.(1)).

The Path-Integral Bosonization analysis proceeds as usually by integrating exactly out the Kalb-Ramond gauge potential field which produces as a result the delta function [3].

$$\begin{aligned} Z[J, \eta, \bar{\eta}] &= \int D^F[H] D^F[\psi] D^F[\bar{\psi}] \delta^{(F)}(\partial_\lambda H^{\lambda\mu\nu} - J^{\mu\nu}) \\ &\times \exp\left\{-\int_{R^4} d^4x \left[\frac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} + \bar{\psi} (i \not{\partial} + ie\gamma^\alpha \gamma^\beta \gamma^\mu H_{\alpha\beta\mu}) \psi \right] (x)\right\}. \end{aligned} \quad (3)$$

Let us note that the delta functional integrand inside of the path integral eq.(3) imposes the classical equations of motion on the three-form Kalb-Ramond strenght H which by its turn can be exactly solved by the Rham-Hodge theorem in terms of the effective dual scalar axion (zero-form) dynamical degree of freedom in the KR theory defined in a space-time topologically trivial as considered in our path integral eq.(3)

$$H_{\lambda\mu\nu} = e\varepsilon^{\lambda\mu\nu\rho} \partial_\rho \vartheta + \partial^{[\lambda} \frac{1}{\partial^2} J^{\mu\nu]}. \quad (4)$$

At this point we re-write the effective action eq.(3) in the previously published author's four-dimensional bosonized chiral action [4]

$$\begin{aligned}
Z[J, \eta, \bar{\eta}] &= \int D^F[\vartheta] \frac{1}{2} \det(-\partial^2) \\
&\times \exp \left\{ -\frac{1}{2} \int_{R^4} d^4x \left[e^2 \partial_\mu \vartheta \partial^\mu \vartheta + \frac{1}{2} J^{\mu\nu} \left(-\frac{1}{\partial^2} \right) J_{\mu\nu} \right] (x) \right\} \\
&\times \int D^F[\psi] D^F[\bar{\psi}] \exp \left\{ -\frac{1}{2} \int_{R^4} d^4x (\bar{\psi} e^{ie\gamma_5 \vartheta} \not{\partial} e^{ie\gamma_5 \vartheta} \psi)(x) \right\} \\
&\times \exp \left\{ -\frac{1}{2} \int_{R^4} d^4x \left(ie \bar{\psi} \left[\gamma^\alpha \gamma^\beta \gamma^\rho \partial^{[\alpha} \frac{1}{\partial^2} J^{\beta\rho]} \right] \right) \psi \right\} (x) \\
&\times \exp \left\{ -i \int_{R^4} d^4x (\psi \bar{\eta} + \bar{\psi} \eta)(x) \right\}. \tag{5}
\end{aligned}$$

After considering the chiral-fermion field variable change on the fermionic path-integral term of eq.(5)

$$\bar{\psi} = \bar{\chi} e^{-ie\gamma_5 \vartheta} \tag{6a}$$

$$\psi = e^{-ie\gamma_5 \vartheta} \chi \tag{6b}$$

$$\begin{aligned}
D[\psi] D[\bar{\psi}] &= D[\chi] D[\bar{\chi}] \frac{\det[e^{ie\gamma_5 \vartheta} \not{\partial} e^{ie\gamma_5 \vartheta}]}{\det[\not{\partial}]} \\
&= D[\chi] D[\bar{\chi}] J[\vartheta], \tag{6c}
\end{aligned}$$

we obtain the exactly bosonized path-integral representation for the KR first order theory as given by eq.(2), namely:

$$\begin{aligned}
Z[J, \eta, \bar{\eta}] &= \int D^F[\vartheta] D[\chi] D[\bar{\chi}] J[\vartheta] \\
&\times \exp \left\{ - \int_{R^4} d^4x \left[\frac{e^2}{2} \partial_\mu \vartheta \partial^\mu \vartheta - \frac{1}{2} J^{\mu\nu} (\partial^2)^{-1} J_{\mu\nu} \right] (x) \right\} \\
&\times \exp \left\{ -\frac{1}{2} \int_{R^4} d^4x (\bar{\chi} \not{\partial} \chi)(x) \right\} \\
&\times \exp \left\{ -\frac{1}{2} ie \int_{R^4} d^4x \left(\bar{\chi} \left(\gamma^\alpha \gamma^\mu \gamma^\nu \partial^{[\alpha} \frac{1}{\partial^2} J^{\mu\nu]} \right) \chi \right) (x) \right\} \\
&\times \exp \left\{ i \int_{R^4} d^4x (\bar{\chi} e^{-ie\gamma_5 \vartheta} \eta + \bar{\eta} e^{-ie\gamma_5 \vartheta}) (x) \right\}, \tag{7}
\end{aligned}$$

here the functional Fermion Jacobian eq.(6c) has been exactly evaluated in refs. [4]:

$$\begin{aligned}
J_\varepsilon[\vartheta] &= \exp \left\{ \frac{e^2}{4\pi^2\varepsilon} \int_{R^4} d^4x (\partial_\mu \vartheta)^2(x) \right\} \\
&\times \exp \left\{ -\frac{e^2}{4\pi^2} \int_{R^4} d^4x (\partial^2 \vartheta)(\partial^2 \vartheta)(x) \right\} \\
&\times \exp \left\{ \frac{e^4}{12\pi^2} \int_{R^4} d^4x [\vartheta(\partial_\mu \vartheta)^2(-\partial^2 \vartheta)](x) \right\}. \tag{8}
\end{aligned}$$

As a first remark to be made on the above written result we note that its first term has the effect of formally inducing a renormalization of the e-charge after the cut-off removing $\varepsilon \rightarrow 0$ on the complete result eq.(2), namely

$$e_{\text{bare}}^2(\varepsilon) \left(1 + \frac{1}{4\pi^2\varepsilon} \right) = e_{\text{ren}}^2. \tag{9}$$

By secondly, we point out the appearance of the fourth-order kinetic term for the scalar effective KR field $\vartheta(x)$, a very important result for the model ultra-violet superrenormalizability.

Another important physical result coming from the set eq.(7)–eq.(9) is the explicitly fermionic matter asymptotic freedom as can be seen directly from the factorized – decoupled form of the full interacting matter fermionic propagator, namely

$$\frac{\delta Z[\eta, \bar{\eta}, J]}{\delta \eta_\alpha(x) \delta \bar{\eta}_\beta(y)} \Big|_{\eta=\bar{\eta}=0}^{J=0} = S_{\alpha\beta}(x-y) \times F(x,y) \tag{10}$$

with $S_{\alpha\beta}(x-y)$ denoting the free fermion propagator and the (decoupled) Kalb-Ramond form factor being given exactly by the (perturbative finite) fourth-order ϑ -path integral as remarked above.

$$\begin{aligned}
F(x,y) &= \int D^F[\vartheta] e^{-\frac{1}{2}e_{\text{ren}}^2 \int_{R^4} (\partial_\mu \vartheta)^2(x) d^4x} \\
&\times e^{-\frac{e_{\text{ren}}^2}{4\pi^2} \int_{R^4} (\partial_\mu^2 \vartheta)^2(x) d^4x} \\
&\times e^{+\frac{e_{\text{ren}}^2}{4\pi^2} \int_{R^4} [\vartheta(\partial_\mu \vartheta)^2(-\partial_\mu^2 \vartheta)](x) d^4x} \\
&\times \{ (\exp -ie_{\text{ren}}\gamma_5 \vartheta(x)) (\exp -ie_{\text{ren}}\gamma_5 \vartheta(y)) \} \tag{11}
\end{aligned}$$

which goes to 1 in the high energy limit of $|x-y| \rightarrow 0$ as a result of the path-integral superrenormalizability associated to the effective axion scalar dual Kalb-Ramond theory eq.(1).

A important calculational remark to be made at this point of our letter is related to the exactly solubility for the Macroscopic radiative corrections of the Kalb-Ramond gauge potential propagator ([3]).

$$\begin{aligned}
& \frac{1}{i^2} \frac{\delta^2[J, \eta, \bar{\eta}]}{\delta J_{\mu\nu}(x) \delta J_{\alpha\beta}(y)} \Big|_{\substack{\eta=\bar{\eta}=0 \\ J=0}} = \langle B_{\mu\nu}(x) B_{\alpha\beta}(y) \rangle \\
& = (-\partial^2)^{-1}(x, y) + e_{\text{ren}}^2 \int d^4z d^4z' (-\partial^2)^{-1}(z-x) (-\partial^2)^{-1}(z-y) \\
& \quad \times \partial_z^{[\lambda} \partial_{z'}^{\lambda']} \langle (\bar{\chi}(z) (\gamma^\lambda \gamma^{[\mu} \gamma^{\nu]}) \chi(z)) (\bar{\chi}(z') (\gamma^{\lambda'} \gamma^{[\alpha} \gamma^{\beta]}) \chi(z')) \rangle^{(0)}, \tag{12}
\end{aligned}$$

here $\langle \rangle^{(0)}$ denotes the freefermion average path integral

$$\langle \rangle^{(0)} = \int D(\chi) D[\bar{\chi}] e^{-\frac{1}{2} \int_{R^4} d^4x (\bar{\chi} \partial x)(x)}. \tag{13}$$

The exactly evaluation of the quantum correction eq.(12) is standard and straightforward by using the well-known Dirac matrixes relationship

$$\gamma^\lambda \gamma^\mu \gamma^\nu = (S_{\lambda\mu\nu\sigma} + \varepsilon_{\lambda\mu\nu\sigma} \gamma_5) \gamma^\sigma \tag{14}$$

$$S_{\lambda\mu\nu\sigma} = (\delta_{\lambda\mu} \delta_{\nu\sigma} + \delta_{\mu\nu} \delta_{\lambda\sigma} - \delta_{\lambda\nu} \delta_{\mu\sigma}). \tag{15}$$

The above exposed results concludes our note about the four-dimensional exactly path-integral Bosonization of our abelian interacting KR field, a new result on the subject.

At this point we call the reader attention that a massive KR field has different numbers of dynamical degrees of freedom from the massless field analyzed in this letter, however leading to a massive quantum electrodynamics as its counterpart for our Four-Dimensional Bosonized path integral eq.(7).

Generaliation of the above results to the non-abelian case and to the presence of back-ground flavor fields, which certainly will taken into account the full axion-scalar quantum path-integral eq.(11), will be presented elsewhere in an extended paper.

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