Bosonization of Massless Kalb-Ramond Fields in the presence of Fermions: A Path-Integral study

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Abstract

We present an exactly soluble, four-dimensional path-integral Bosonization procedure for dynamical fermions interacting through a spin-orbit coupling with abelian Kalb-Ramond Gauge fields.

In the last years, Kalb-Ramond field theory has been widely studied as an alternative dynamical quantum field scheme to the Higgs mechanism [1], as well as in relation to the dynamics of strings in the problem of string repesentation for Q.C.D. at large number of colors [2]. The basic formalism used to analyze such Kalb-Ramond non-perturbative quantum dynamics has been the Path-integral formalism, which has shown itself to be a very powerful procedure to understand correctly the differente phases of the associated Kalb-Ramond Quantum Field Theory [3].

One important problem in those Path-integral studies, still missing in the literature, is that one related to the presence of interacting dynamical fermions in the Kalb-Ramond Gauge theory. In this letter, we shall describe the extension of previous path-integral dualization-bosonization studies [3] to the case of Fermionic matter coupling through a spin-orbit field quantum interaction.

Let us start by considering the Abelian Kalb-Ramond first order action but now in the presence of dynamical fermions in the four-dimensional Euclidean world.

$$S[H,B,\psi,\overline{\psi}] = \int_{R^4} d^4x \left\{ \frac{1}{12} H_{\lambda\mu\nu}H^{\lambda\mu\nu} - \frac{1}{6} H^{\lambda\mu\nu}\partial_{[\lambda}B_{\mu\nu]} + \overline{\psi}(i\ \partial\!\!\!/ + ie\gamma^\alpha\gamma^\beta\gamma^\mu H_{\alpha\beta\mu})\psi \right\}. \quad (1)$$

Here the dynamical fields are the independent three-form H, the KR gauge field B and the Dirac fermion fields $(\psi,\overline{\psi})$.

We shall apply the bosonization procedure in the path-integral framework through the following theory's generating functional (normalized to unity) ([4]).

$$\begin{aligned} Z[J,\eta,\overline{\eta}] = & \int D^F[H]D^F[B]D^F[\psi]D^F[\overline{\psi}] \\ & \times \exp\{-S[H,B,\psi,\overline{\psi}]\} \\ & \times \exp\left\{-\frac{i}{2}\int_{R^4} d^4x(\overline{\eta}\psi + \overline{\psi}\eta + JB)(x)\right\}. \end{aligned} \quad (2)$$

It is worth call the reader attention that the Path-integral eq.(2) is invariant under the KR gauge symmetry, provide the external source corrent $J_{\mu\nu}$ is chosen to be divergence free and due to our proposed action term related to the direct interaction of the quantum fermionic matter with the Kalb-Ramond gauge field through its strenght three-form H – the spin orbit fermion interaction. (see eq.(1)).

The Path-Integral Bosonization analysis proceeds as usually by integrating exactly out the Kalb-Ramond gauge potential field which produces as a result the delta function [3].

$$\begin{aligned} Z[J,\eta,\overline{\eta}] = & \int D^F[H]D^F]\psi]D^F[\overline{\psi}]\delta^{(F)}(\partial_\lambda H^{\lambda\mu\nu} - J^{\mu\nu}) \\ & \times \exp\left\{-\int_{R^4} d^4x \left[\frac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} + \overline{\psi}(i\ \partial\!\!\!/ + ie\gamma^\alpha\gamma^\beta\gamma^\mu H_{\alpha\beta\mu})\psi\right](x)\right\}. \end{aligned} \quad (3)$$

Let us note that the delta functional integrand inside of the path integral eq.(3) imposes the classical equations of motion on the three-form Kalb-Ramond streght H which by its turn can be exactly solved by the Rham-Hodge theorem in terms of the effective dual scalar axion (zero-form) dynamical degree of freedom in the KR theory defined in a space-time topologically trivial as considered in our path integral eq.(3)

$$H_{\lambda\mu\nu} = e\varepsilon^{\lambda\mu\nu\rho}\partial_\rho\vartheta + \partial^{[\lambda}\frac{1}{\partial^2}J^{\mu\nu]}. \quad (4)$$

At this point we re-write the effective action eq.(3) in the previously published author's four-dimensional bosonized chiral action [4]

$$
\begin{aligned}
Z[J,\eta,\overline{\eta}] = & \int D^F[\vartheta]\frac{1}{2}\det(-\partial^2) \\
& \times \exp\left\{-\frac{1}{2}\int_{R^4} d^4x\left[e^2\partial_\mu\vartheta\partial^\mu\vartheta+\frac{1}{2}J^{\mu\nu}\left(-\frac{1}{\partial^2}\right)J_{\mu\nu}\right](x)\right\} \\
& \times \int D^F[\psi]D^F[\overline{\psi}]\exp\left\{-\frac{1}{2}\int_{R^4} d^4x(\overline{\psi}e^{ie\gamma_5\vartheta}\ \partial\!\!\!/ e^{ie\gamma_5\vartheta}\psi)(x)\right\} \\
& \times \exp\left\{-\frac{1}{2}\int_{R^4} d^4x\left(ie\overline{\psi}\left[\gamma^\alpha\gamma^\beta\gamma^\rho\partial^{[\alpha}\frac{1}{\partial^2}J^{\beta\rho]}\right]\right)\psi\right\}(x) \\
& \times \exp\left\{-i\int_{R^4} d^4x(\psi\overline{\eta}+\overline{\psi}\eta)(x)\right\}.
\end{aligned} \tag{5}
$$

After considering the chiral-fermion field variable change on the fermionic path-integral term of eq.(5)

$$\overline{\psi} = \overline{\chi}e^{-ie\gamma_5\vartheta} \tag{6a}$$

$$\psi = e^{-ie\gamma_5\vartheta}\chi \tag{6b}$$

$$
\begin{aligned}
D[\psi]D[\overline{\psi}] &= D[\chi]D[\overline{\chi}]\frac{\det[e^{ie\gamma_5\vartheta}\ \partial\!\!\!/ e^{ie\gamma_5\vartheta}]}{\det[\partial\!\!\!/]} \\
&= D[\chi]D[\overline{\chi}]J[\vartheta],
\end{aligned} \tag{6c}
$$

we obtain the exactly bosonized path-integral representation for the KR first order theory as given by eq.(2), namely:

$$
\begin{aligned}
Z[J,\eta,\overline{\eta}] = & \int D^F[\vartheta]D[\chi]D[\overline{\chi}]J[\vartheta] \\
& \times \exp\left\{-\int_{R^4} d^4x\left[\frac{e^2}{2}\partial_\mu\vartheta\partial^\mu\vartheta-\frac{1}{2}J^{\mu\nu}(\partial^2)^{-1}J_{\mu\nu}\right](x)\right\} \\
& \times \exp\left\{-\frac{1}{2}\int_{R^4} d^4x(\overline{\chi}\ \partial\!\!\!/\chi)(x)\right\} \\
& \times \exp\left\{-\frac{1}{2}ie\int_{R^4} d^4x\left(\overline{\chi}\left(\gamma^\alpha\gamma^\mu\gamma^\nu\partial^{[\alpha}\frac{1}{\partial^2}J^{\mu\nu]}\right]\chi\right)(x)\right\} \\
& \times \exp\left\{i\int_{R^4} d^4x\left(\overline{\chi}e^{-ie\gamma_5\vartheta}\eta+\overline{\eta}e^{-ie\gamma_5\vartheta}\right)(x)\right\},
\end{aligned} \tag{7}
$$

here the functional Fermion Jacobian eq.(6c) has been exactly evaluated in refs. [4]:

$$
\begin{aligned}
J_\varepsilon[\vartheta] = \exp & \left\{ \frac{e^2}{4\pi^2\varepsilon} \int_{R^4} d^4x (\partial_\mu \vartheta)^2(x) \right\} \\
& \times \exp \left\{ -\frac{e^2}{4\pi^2} \int_{R^4} d^4x (\partial^2\vartheta)(\partial^2\vartheta)(x) \right\} \\
& \times \exp \left\{ \frac{e^4}{12\pi^2} \int_{R^4} d^4x [\vartheta(\partial_\mu\vartheta)^2(-\partial^2\vartheta)](x) \right\}. \qquad (8)
\end{aligned}
$$

As a first remark to be made on the above written result we note that it first term has the effect of formally inducing a renormaliation of the e-charge after the cutt-off removing $\varepsilon \to 0$ on the complete result eq.(2), namely

$$
e^2_{\text{bare}}(\varepsilon) \left(1 + \frac{1}{4\pi^2\varepsilon} \right) = e^2_{\text{ren}}. \qquad (9)
$$

By secondly, we point out the appearance of the fourth-order kinetic term for the scalar effective KR field $\vartheta(x)$, a very important result for the model ultra-violet superenormalizability.

An another important physical result coming from the set eq.(7)–eq.(9) is the explicitly fermionic matter asymptotic freedom as can be see directly from the factorized – decoupled form of the full interacting matter fermionic propagator, namely

$$
\left. \frac{\delta Z[\eta, \overline{\eta}, J]}{\delta\eta_\alpha(x)\delta\eta_\beta(y)} \right|_{\substack{J=0 \\ \eta=\overline{\eta}=0}} = S_{\alpha\beta}(x-y) \times F(x,y) \qquad (10)
$$

with $S_{\alpha\beta}(x-y)$ denoting the free fermion propagator and the (decoupled) Kalb-Ramond form factor being given exactly by the (perturbative finite) fourth-order ϑ-path integral as remarked above.

$$
\begin{aligned}
F(x,y) = \int & D^F[\vartheta] e^{-\frac{1}{2} e^2_{\text{ren}} \int_{R^4} (\partial_\mu\vartheta)^2(x) d^4x} \\
& \times e^{-\frac{e^2_{\text{ren}}}{4\pi^2} \int_{R^4} (\partial^2_\mu \vartheta)^2(x) d^4x} \\
& \times e^{+\frac{e^2_{\text{ren}}}{4\pi^2} \int_{R^4} [\vartheta(\partial_\mu\vartheta)^2(-\partial^2_\mu\vartheta)](x) d^4x} \\
& \times \{(\exp -ie_{\text{ren}}\gamma_5\vartheta(x))(\exp -ie_{\text{ren}}\gamma_5\vartheta(y)\} \qquad (11)
\end{aligned}
$$

which goes to 1 in the high energy limit of $|x-y| \to 0$ as a result of the path-integral super renormalizability associated to the effective axion scalar dual Kalb-Ramond theory eq.(1)).

A important calculational remark to be made at this point of our letter is related to the exactly solubility for the Macroscopic radiative corrections of the Kalb-Ramond gauge potential propagator ([3]).

$$\frac{1}{i^2}\frac{\delta^2[J,\eta,\overline{\eta}]}{\delta J_{\mu\nu}(x)\delta J_{\alpha\beta}(y)}\Big|_{\substack{\eta-\overline{\eta}=0\\ J=0}} = \langle B_{\mu\nu}(x)B_{\alpha\beta}(y)\rangle$$
$$= (-\partial^2)^{-1}(x,y) + e^2_{\text{ren}}\int d^4z d^4z'(-\partial^2)^{-1}(z-x)(-\partial^2)^{-1}(z-y)$$
$$\times\, \partial_z^{[\lambda}\partial_{z'}^{\lambda'}\langle(\overline{\chi}(z)(\gamma^\lambda\gamma^{[\mu}\gamma^{\nu]})\chi(z))(\overline{\chi}(z')(\gamma^{\lambda'}\gamma^{[\alpha}\gamma^{\beta]})\chi(z'))\rangle^{(0)}, \tag{12}$$

here $\langle\ \rangle^{(0)}$ denotes the freefermion average path integral

$$\langle\ \rangle^{(0)} = \int D(\chi)D[\overline{\chi}]e^{-\frac{1}{2}\int_{R^4}d^4x(\overline{\chi}\partial\!\!\!/ x)(x)}. \tag{13}$$

The exactly evaluation of the quantum correction eq.(12) is standard and straightforward by using the well-known Dirac matrixes relationship

$$\gamma^\lambda\gamma^\mu\gamma^\nu = (S_{\lambda\mu\nu\sigma} + \varepsilon_{\lambda\mu\nu\sigma}\gamma_5)\gamma^\sigma \tag{14}$$

$$S_{\lambda\mu\nu\sigma} = (\delta_{\lambda\mu}\delta_{\nu\sigma} + \delta_{\mu\nu}\delta_{\lambda\sigma} - \delta_{\lambda\nu}\delta_{\mu\sigma}). \tag{15}$$

The above exposed results concludes our note about the four-dimensional exactly path-integral Bosonization of our abelian interacting KR field, a new result on the subject.

At this point we call the reader attention that a massive KR field has different numbers of dynamical degrees of freedom from the massless field analyzed in this letter, however leading to a massive quantum electrodynamics as its counterpart for our Four-Dimensional Bosonized path integral eq.(7).

Generaliation of the above results to the non-abelian case and to the presence of back-ground flavor fields, which certainly will taken into account the full axion-scalar quantum path-integral eq.(11), will be presented elsewhere in an extended paper.

Acknowledgments: We are thankfull to UFF/Niterói - Rio de Janeiro.

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